Black Holes and Quantum Mechanics

Juan Maldacena

Institute for Advanced Study

Karl Schwarzschild Meeting
Frankfurt Institute for Advanced Study, 2017
Letter from Schwarzschild to Einstein.

22 December 1915,

\[ ds^2 = \left(1 - \frac{\gamma}{R}\right) dt^2 - \frac{dR^2}{(1 - \frac{\gamma}{R})} - R^2 (d\theta^2 + \sin^2 \theta d\phi^2) \]
It as been very confusing ever since...
Classically...
\[ ds^2 = \left(1 - \frac{\gamma}{R}\right) dt^2 - \frac{dR^2}{(1 - \frac{\gamma}{R})} - R^2 (d\theta^2 + \sin^2 \theta d\phi^2) \]

Extreme slow down of time
The coordinate ``singularity''

Eddington 1924 finds a non singular coordinate system but did not recognize (or comment on its significance).

Lemaitre 1933 First published statement that the horizon is not singular.

Einstein Rosen 1935 (Still call it a ``singularity'')

Szekeres, Kruskal 59-60 coordinates cover the full spacetime

Wheeler Fulling 62 It is a wormhole!

Future singularity

Oppenheimer Snyder 1939 Observer on the surface does not feel anything special at the horizon. Finite time to the singularity
It took 45 years to understand a classical solution... Why?
The symmetries are realized in a funny way.
The time translation symmetry $\rightarrow$ boost symmetry at the bifurcation surface.

Future singularity

Interior is time dependent for an observer falling in. It looks like a big crunch.
Black holes in astrophysics

• Quasars  (most efficient energy sources)

• Stellar mass black holes

• Sources for gravity waves !

• Our own supermassive black hole in the Milky Way...
Quantum gravity is necessary at the singularity!

What signals?

Singularity is behind the horizon.

It is shielded behind the black hole horizon that acts as a Schwarz-Shild.
Black holes and quantum mechanics

• Black holes are one of the most surprising predictions of general relativity.
• Incorporating quantum mechanics leads to a new surprise:

Black holes are not black!
Hot black holes

- Black holes have a temperature
  \[ T \sim \frac{\hbar}{r_H} \]

- An accelerated expanding universe also has a temperature
  \[ T \sim \frac{\hbar}{R_H} \]

We can have white "black holes"

Hawking

Very relevant for us!

Chernikov, Tagirov, Figari, Hoegh-Krohn, Nappi, Gibbons, Hawking, Bunch, Davies, ....
Quantum mechanics is crucial for understanding the large scale geometry of the universe.
Why a temperature?

- Consequence of special relativity + quantum mechanics.
Flat space first
Why a temperature?

Accelerated observer $\rightarrow$ energy = boost generator.
Time translation $\rightarrow$ boost transformation
Continue to Euclidean space $\rightarrow$ boost becomes rotation.
Why a temperature?

Continue to Euclidean space $\rightarrow$ boost becomes rotation.

Angle is periodic $\rightarrow$ temperature

$$\beta = \frac{1}{T} = 2\pi r = \frac{2\pi}{a}$$

Ordinary accelerations are very small, $g = 9.8 \text{ m/s}^2 \rightarrow \beta = 1 \text{ light year}$

Bisognano Weichman, Unruh

Thermische Unruhe
Horizon: accelerated observer only has access to the right wedge.

If we only make observations on the right wedge \(\rightarrow\) do not see the whole system \(\rightarrow\) get a mixed state (finite temperature).

General prediction, only special relativity + quantum mechanics + locality
Vacuum is highly entangled!
Black hole from collapse

Black hole case

Equivalence principle: region near the horizon is similar to flat space.

If we stay outside $\rightarrow$ accelerated observer $\rightarrow$ temperature.
Black holes have a temperature

Do they obey the laws of thermodynamics?
Black hole entropy

\[ T \sim \frac{\hbar}{r_H} \]

Special relativity near the black hole horizon

\[ r_H \leftrightarrow M \]

Einstein equations

\[ dM = TdS \]

1\textsuperscript{st} Law of thermodynamics

\[ S = \frac{(\text{Area})}{4\hbar G_N} \]

Black hole entropy

Bekenstein, Hawking

2\textsuperscript{nd} Law \(\rightarrow\) area increase from Einstein equations and positive null energy condition. Hawking
Including the quantum effects

\[ S_{BH} = \frac{(\text{Area})}{4G_N} + S_{\text{entanglement}} \]

Entanglement entropy of quantum fields across the black hole horizon

Has been understood better in quantum field theory

2\textsuperscript{nd} Law extended to include this term.

Bekenstein bound $\rightarrow$ automatic in relativistic quantum field theory.

Focusing theorems and better understanding of the positivity of energy, and new ``area'' increase statements.

Wall 2011

Casini 2008

Bousso’s talk
Bousso, Englehardt, Wall, Faulkner, ...
Bekenstein – Casini bound

\[ S \leq 2\pi ER \]

When is this true? Is it true? Does it impose a constraint on QFT?

\[ \Delta S \leq 2\pi \Delta E_{\text{Rindler}} \]

It is always true in any relativistic QFT.

2nd Law always satisfied.
General relativity and thermodynamics
General relativity and thermodynamics

• Black hole seen from the outside = thermal system with finite entropy.

• Is there an exact description where information is preserved?

• Yes...
Gauge/Gravity Duality
(or gauge/string duality, AdS/CFT, holography)

Theories of quantum interacting particles
(very strongly interacting)

Quantum dynamical Space-time
(General relativity)
string theory

JM 97
Witten, Gubser, Polyakov, Klebanov
....
Gravity in asymptotically Anti de Sitter Space

Duality

Gravity, Strings

Quantum interacting particles, quantum field theory
Black holes in a gravity box

Hot fluid made out of very strongly interacting particles.
Lessons for black holes

• Black holes as seen from outside (from infinity) are like an ordinary quantum system.
• Black hole entropy = ordinary entropy of the quantum system.
• Absorption into the black hole = thermalization
• Chaos $\rightarrow$ near horizon gravity

• Interior ?
In the meantime
Black holes as a source of information

Black holes as toy models

Used to model strongly interacting systems in high energy physics or condensed matter physics.

Hot fluid made out of strongly interacting particles.

Key insights into the theory of hydrodynamics with anomalies.

Damour, Herzog, Son, Kovtun, Starinets, Bhattacharyya, Hubeny, Loganayagam, Mandal, Minwalla, Morita, Rangamani, Reall, Bredberg, Keeler, Lysov, Strominger...
Let us go back to chaos
Chaos $\rightarrow$ divergence of nearby trajectories

Thermal system $\rightarrow$ average over all trajectories

Growth $\rightarrow$ Where you are after the perturbation vs. where you would have been.
Classical

\[ \frac{\delta q(t)}{\delta q(0)} = \{q(t), p(0)\} , \quad \{q(t), p(0)\}^2 \]

\[ \delta q(t) \propto e^{\lambda t} \]

Quantum

\[ [Q(t), P(0)]^2 \]

Quantum General:

\[ \langle [W(t), V(0)]^2 \rangle_\beta \propto \frac{1}{N} e^{\lambda t} \]

\( W, V \) are two ``simple'' (initially commuting) observables.
Imagine we have a large N system. This is the definition of the quantum Liapunov exponent.
For quantum systems that have a gravity dual

Commutator $\rightarrow$ involves the scattering amplitude between these two excitations.

Leading order $\rightarrow$ graviton exchange

Large $t$ $\rightarrow$ large boost between the two particles.

Gravitational interaction has spin 2, Shapiro time delay proportional to energy.

Energy goes as $e^t$

$$\langle [W(t), V(0)]^2 \rangle_{\beta} \propto \frac{1}{N} e^{\frac{2\pi}{\beta} t}$$

$$\lambda = \frac{2\pi}{\beta} = 2\pi T$$
Can it be different?

Graviton → phase shift: \( \delta(s) \sim G_N s \rightarrow \lambda = \frac{2\pi}{\beta} \)

String → phase shift

\[ \delta(s) \sim G_N s^{1+\alpha'} t \rightarrow \lambda = \frac{2\pi}{\beta} \left(1 - \frac{l_s^2}{R^2}\right) \]

s, t = Mandelstam invariants

Typical size of string (of graviton in string theory)

Radius of curvature of black hole
It can be less...

More ?

In flat space a phase shift has to scale with a power of $s$ less than one in order to have a causal theory

Maybe there is a bound...
Black holes as the most chaotic systems

\[ \lambda \leq \frac{2\pi}{\beta} = 2\pi T \]

For any large N (small hbar) quantum system.

(Strings connect weakly coupled to strongly coupled systems)

Sekino Susskind
JM, Shenker, Stanford
How do we get order from chaos?

How do we get the vacuum from chaos, or from a chaotic quantum system?

Example: hydrodynamics, we get something simple for some interactions, but it is more complicated with very small interactions (Boltzmann equation).
The full Schwarzschild wormhole

No need to postulate any exotic matter

No matter at all!
View it as an entangled state

\[ |TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R \]
What is this funny “time translation symmetry”? 

\[ U = e^{it(H_R - H_L)} \]

**Exact symmetry**

**Exact boost symmetry!**

\[
|TFD\rangle = \sum_n e^{-\beta E_n/2} \bar{E}_n \rangle_L |E_n\rangle_R
\]
If Bob sends a signal, then Alice cannot receive it.

These wormholes are not traversable.

Relay on Integrated null energy condition (Now a theorem, proven using entanglement methods) Balakrishnan, Faulkner, Khandker, Wang

Not good for science fiction.

Good for science!

\[ |TFD\rangle = \sum_n e^{-\beta E_n/2} |\overline{E_n}\rangle_L |E_n\rangle_R \]
Interior is common

If they jump in, they can meet in the interior!

But they cannot tell anyone.

\[ |TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R \]
Spacetime connectivity from entanglement

\[ \text{ER} = \text{EPR} \]

Van Raamsdonk
Verlinde\(^2\)
Papadodimas Raju
JM Susskind
Entanglement and geometry
Local boundary quantum bits are highly interacting and very entangled.

\[ S(R) = \frac{A_{\text{min}}}{4G_N} \]

Ryu, Takayanagi, Hubeny, Rangamani

Generalization of the Black hole entropy formula.
Interesting connections to quantum information theory

Quantum error correction  Almheiri, Dong, Harlow, Preskill, Yoshida, Pastawski

Complexity theory  Harlow, Hayden, Brown, Susskind
Conclusions

• Black holes are extreme objects: most compact, most efficient energy conversion, most entropic, most chaotic,...

• Most confusing...

• The process of unravelling these confusions has lead to better understand of gravity, quantum systems, string theory and their interconnections.

• Black holes are not only in the cosmos, but can also be present in the lab.

• And there are still very important confusions and open problems: Interior and singularity ?
Thank you!
Extra slides
Full Schwarzschild solution

Simplest spherically symmetric solution of pure Einstein gravity (with no matter)

Eddington, Lemaitre, Einstein, Rosen, Finkelstein, Kruskal
Wormhole interpretation.
Wormhole interpretation.

Note: If you find two black holes in nature, produced by gravitational collapse, they will not be described by this geometry.
No faster than light travel

Figure y: Maximally extended Schwarzschild spacetime. There are two asymptotic regions. The blue spatial slice contains the Einsteint-Rosen bridge connecting the two regions that are not in causal contact and information cannot be transmitted across the bridge. This can easily be seen from the Penrose diagrams and is consistent with the fact that entanglement does not imply nontlocal signal propagation.

(a) (b)

Figure z: Another representation of the blue spatial slice of figure y. It contains a neck connecting two asymptotically flat regions. Here we have two distant entangled black holes in the same space. The horizons are identified as indicated. This is not an exact solution of the equations but an approximate solution where we can ignore the small force between the black holes.

All of this is well known, but what may be less familiar is a third interpretation of the eternal Schwarzschild black hole. Instead of black holes on two disconnected sheets, we can consider two very distant black holes in the same space. If the black holes were not entangled we would not connect them by an Einsteint-Rosen bridge. But if they are somehow created at the same time, the Penrose diagrams represent the entanglement. Of course, in this case the dynamical decoupling is not exact.

Fuller, Wheeler, Friedman, Schleich, Witt, Galloway, Wooglar
In the exact theory, each black hole is described by a set of microstates from the outside.

Wormhole is an entangled state

\[ |\Psi\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L \times |E_n\rangle_R \]

Geometric connection from entanglement. \( ER = EPR \)
Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. Einstein, B. Podolsky and N. Rosen, Institute for Advanced Study, Princeton, New Jersey
(Received March 25, 1935)

EPR

The Particle Problem in the General Theory of Relativity

A. Einstein and N. Rosen, Institute for Advanced Study, Princeton
(Received May 8, 1935)
A forbidden meeting

Figure y: Maximally extended Schwarzschild spacetime. There are two asymptotic regions. The blue spatial slice contains the Einstein-Rosen bridge connecting the two regions. This can easily be seen from the Penrose diagrams and is consistent with the fact that entanglement does not imply nonlocal signal propagation.

(a) Figure z: Another representation of the blue spatial slice of figure y. It contains a neck connecting two asymptotically flat regions. Here we have two distant entangled black holes in the same space. The horizons are identified as indicated. This is not an exact solution of the equations but an approximate solution where we can ignore the small force between the black holes.

All of this is well known. But what may be less familiar is a third interpretation of the eternal Schwarzschild black hole. Instead of black holes on two disconnected sheets, we can consider two very distant black holes in the same space. If the black holes were not entangled we would not connect them by a Einstein-Rosen bridge. But if they are somehow created at $t = 0$ with entangled state, they represent the entanglement. See figure zobpu. Of course in this case the dynamical decoupling is not valid.
Figure y: Maximally extended Schwarzschild spacetime. There are two asymptotic regions. The blue spatial slice contains the Einsteint-Rosen bridge connecting the two regions that are not in causal contact and information cannot be transmitted across the bridge. This can easily be seen from the Penrose diagrams and is consistent with the fact that entanglement does not imply nontlocal signal propagation.

Figure z: Another representation of the blue spatial slice of figure y. It contains a neck connecting two asymptotically flat regions. Here we have two distant entangled black holes in the same space. The horizons are identified as indicated. This is not an exact solution of the equations but an approximate solution where we can ignore the small force between the black holes.

All of this is well known, but what may be less familiar is a third interpretation of the eternal Schwarzschild black hole. Instead of black holes on two disconnected sheets, we can consider two very distant black holes in the same space. If the black holes were not entangled, we would not connect them by a Einsteint-Rosen bridge. But if they are somehow created at the same time with entangled state, then the bridge between them represents the entanglement. Of course, in this case, the dynamical decoupling is not...
String theory

• String theory started out defined as a perturbative expansion.
• String theory contains interesting solitons: D-branes.
• Using D-branes one can "count" the number of states of extremal charged black holes in certain superstring theories.
• D-branes inspired some non-perturbative definitions of the theory in some cases.

Matrix theory: Banks, Fischler, Shenker, Susskind
Gauge/gravity duality: JM, Gubser, Klebanov, Polyakov, Witten

Polchinski
Strominger Vafa


Entanglement and geometry

• The entanglement pattern present in the state of the boundary theory can translate into geometrical features of the interior.

• Spacetime is closely connected to the entanglement properties of the fundamental degrees of freedom.

• Slogan: Entanglement is the glue that holds spacetime together...

• Spacetime is the hydrodynamics of entanglement.

Van Raamsdonk
Questions

• Black holes look like ordinary thermal systems if we look at them from the outside. We even have some conjectured exact descriptions.

• How do we describe the interior within the same framework that we describe the exterior?

  Modern version of the information paradox; Mathur, Almheiri, Marolf, Polchinski, Stanford, Sully,..

• Once we figure it out: what is the singularity?

• What lessons do we learn for cosmology?