Strong energy condition and action growth rate bound in holographic complexity

Run-Qiu Yang
Korea Institute for Advanced Study

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Quantum information theory is a good lens to consider the quantum gravity.

Any computation task can be regarded as a transition from one state to an other state:

\[
|0010011010\cdots\rangle \quad \rightarrow \quad |1010110010\cdots\rangle
\]

Complexity can describe the difficulty of any computation task.
To understand the complexity, let’s consider a quantum circuit.

A quantum circuit (QC) is a device composed of qubits and gates (called g) whose purpose is to implement special unitary transformations on an initial state of the qubits.

$$|0_i\rangle = |\uparrow_i\rangle \xrightarrow{g_i} |1_i\rangle = |\downarrow_i\rangle$$

For example, considering a K-spin system:

$$|101011\cdots 1\rangle = g_k \cdots g_6 g_5 g_3 g_1 |0000\cdots 0\rangle = U(n) |0000\cdots 0\rangle$$

U(n) is a element in SU(2^K). The quantum gate complexity is **minimal** gates required to obtain U(n) from I.
The left side shows a discrete path induced by a series of gates. The right side shows a curve induced by a generator.

Define the complexity of an operator

Define the length in the Lie group and find the geodesic

\[
\frac{du}{ds} = -ih(s)u
\]

• **Computers are physical systems:** the laws of physics dictate what they can and cannot do.

• For a compute with energy (mass) $E$, the times it can compute in per second is,

$$\frac{dC}{dt} \leq \frac{2E}{\pi \hbar}$$

• For a compute with mass = 1 kg, the fastest computational ability is about $5 \times 10^{50}$ /sec

*Nature 406, 1047 (2000)*
The Complexity-action (CA) conjecture states that the complexity is given by the bulk action evaluated on the Wheeler-deWitt patch attached at some boundary time $t$.

\[ C = \frac{\mathcal{A}}{\pi \hbar} \]

Quantum machines gives following inequality

\[ \frac{dC}{dt} \leq \frac{2E}{\pi \hbar} \]

CA conjecture shows that the complexity is given by bulk action

\[ C = \frac{A}{\pi \hbar} \]

In asymptotic AdS black hole, E is just the mass of black hole

In late time limit, Schwarzschild AdS black hole saturates this inequality!

A. R. Brown, et. al, Phys. Rev. Lett. 116, 191301 (2016);
Analogue with black hole thermodynamics

Conjectures by quantum gravity

Thermodynamic results

Classical version

T is uniform in static case

κ is uniform in event horizon

T = κ/2π

T can’t reduce to zero by finite operators

κ can’t reduce to zero with finite advanced time

S = A/2π

δS ≥ 0

δA ≥ 0

U = M

δU = TδS + F^i δx^i

δM = κδS/8π + F^i δx^i

Can a similar thing happen in CA conjecture?

Classical version

\[ \frac{dC}{dt} \leq \frac{2E}{\pi \hbar} \]

\[ C = \frac{A}{\pi \hbar} \]

\[ \frac{dA}{dt} \leq 2M \]

Analogue with black hole thermodynamics
In order to answer this question, let’s first pay more attention on the bound equation it self.

\[ \frac{dA}{dt} \leq 2M \]

Has proper definition

Coordinate dependent

Has several different definitions such as: ADM mass, Misner-Sharp mass, Komar mass, Brown-York tensor and so on.

In static case, a good choice for \( t \) is that it is the orbit of Killing vector

\[ \xi^\mu = \nabla_\nu \omega^\nu_{\mu} \]

Killing potential:

\[ m = -\frac{1}{8\pi} \oint_S \left( \nabla^\mu \xi_\nu - \frac{3}{\ell_{\text{AdS}}^2} \omega^\mu_{\nu} \right) dS_{\mu\nu} \]

\[ M = -\frac{1}{8\pi} \int_{S \rightarrow \infty} \left( \nabla^\mu \xi_\nu - \frac{3}{\ell_{\text{AdS}}^2} \omega^\mu_{\nu} \right) dS_{\mu\nu} \]

NOTE: Komar mass may be different from the ADM mass and the mass defined by Brown-York tensor.
For a static asymptotic isolated AdS black hole (maximal extension), if it satisfies that,

- It has topology $\mathbb{R}^2 \times S^2$ and only space-like singularities;
- There is a connected bifurcate Killing horizon;
- Matter fields only distribute in the outside of the Killing horizon;
- Strong energy condition (SEC) is satisfied.

Then at late time limit, this inequality is true,

$$\frac{dA}{dt} \leq 2M$$

As the space-time is static, the action growth in the outside of horizon is canceled with each other

\[ \frac{dA}{dt} = \frac{dA_{\text{in,bulk}}}{dt} + \frac{dA_{\text{in,bd}}}{dt} + \frac{dA_{\text{in,corner}}}{dt} = 2m_H \]

The mass and \( m_H \) is associated by

\[ m(r) = m_H + 2 \int_{\Sigma(r>r_0)} (T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu}) n^\mu \xi^\nu dV \]

\[ m_H \leq M \]

\[ \frac{dA}{dt} \leq 2M \]
• It doesn’t assume any additional symmetry (such as spherical symmetry) in the outside of horizon;

• Although the inequality is proposed by the consideration of CA conjecture, the proof of doesn't rely on the correctness of CA conjecture and holographic duality.

• It is a very strong evidence for CA conjecture!

• It is also interesting to investigate if they could be weakened.

Summary