Loop effects in QED in space with a black hole

Slava Emelyanov

Institute of Theoretical Physics
Karlsruhe Institute of Technology

viacheslav.emelyanov@kit.edu

Frankfurt Institute for Advanced Studies
Frankfurt, 27th July 2017
Overview

Schwarzschild black hole with quantum fields
- Schwarzschild-black-hole evaporation
- Fermion- and vector-field propagator

Radiative corrections in QED
- Photon self-energy
- Electron/positron self-energy

One-loop effects induced by small black holes
- How small?
  - Reminder: Hot neutral $e^-e^+$ plasma
  - Debye-like screening
  - Modified photon dispersion relation
  - Modified electron dispersion relation

Concluding remarks
Schwarzschild black hole with quantum fields

Sketch of Schwarzschild-black-hole evaporation

\[ R_c \sim 3M \]

“Black-hole atmosphere”

Outward-positive-energy flow

Inward-negative-energy flow

Schwarzschild black hole with quantum fields

- Fermion- and vector-field propagator

Black hole evaporate through quantum fields

\[ \uparrow \]

Field propagators get a correction describing the energy flux
Schwarzschild black hole with quantum fields

- **Fermion- and vector-field propagator: Electron/positron field**

  Under the gravitational collapse

  Fermion propagator in near- & far-horizon region becomes

  \[
  \frac{i(\not{p}+m)}{p^2-m^2+i\epsilon} \quad \rightarrow \quad \left[ \frac{i(\not{p}+m)}{p^2-m^2+i\epsilon} - 4\pi^2 g_R \frac{\not{p} \cdot (\not{p}+m)}{e^\beta |p_0| + 1} \delta(p-(p_0^2-m^2)^{1/2}n) \right]
  \]

  *empty space* → *space with a black hole*
Schwarzschild black hole with quantum fields

Fermion- and vector-field propagator: Photon field

Under the gravitational collapse

\[ \frac{-i\eta_{\mu\nu}}{k^2 + i\varepsilon} \quad \Rightarrow \quad \frac{-i\eta_{\mu\nu}}{k^2 + i\varepsilon} - 4\pi^2 g_R \frac{|k|\eta_{\mu\nu}}{e^{\beta|k_0|} - 1} \delta(k - k_0 n) \]

empty space \quad \Rightarrow \quad \text{space with a black hole}
The parameters $g_R$ and $\beta$ can be shown to be given by

$$g_R \approx \frac{27 r_H^2}{16} \begin{cases} +1/R^2 , & R \gg r_H , \\ -4/r_H^2 , & R \sim r_H , \end{cases}$$

and

$$\beta = \frac{1}{T_H} \begin{cases} 1 , & R \gg r_H , \\ 2 , & R \sim r_H , \end{cases}$$

where $T_H$ is the Hawking temperature parameter.
Radiative corrections in QED

**Vacuum polarization tensor**

The classical Maxwell equation changes due to radiative corrections. In momentum space, it reads

\[
\left[ k^2 \eta^{\mu\nu} - k^\mu k^\nu + \Pi^{\mu\nu}(k) \right] A_\nu(k) = 0,
\]

where vacuum polarization tensor is pictorially given by

\[
i \Pi^{\mu\nu}(k) = \begin{array}{c}
\mu \\
\nu \\
k
\end{array}
\]

\[
+ \begin{array}{c}
\mu \\
\nu \\
k
\end{array}
\]

\[
+ 2 \begin{array}{c}
\mu \\
\nu \\
k
\end{array}
\]

\[+ \mathcal{O}(\alpha^3)\]
Radiative corrections in QED

Vacuum polarization tensor

The classical Maxwell equation changes due to radiative corrections. In momentum space, it reads

$$\left[ k^2 \eta^{\mu\nu} - k^\mu k^\nu + \Pi^{\mu\nu}(k) \right] A_\nu(k) = 0,$$

where vacuum polarization tensor is pictorially given by

$$i\Pi^{\mu\nu}(k) = \frac{\mu}{k} \nu$$

$$+ \frac{\mu}{k} \nu + 2 \frac{\mu}{k} \nu + O(\alpha^2)$$
The vacuum polarization tensor can be represented as follows:

\[ \Pi^{\mu\nu}(k) = \pi_T(k_0, k) P^{\mu\nu} + \pi_L(k_0, k) Q^{\mu\nu}, \]

where \( P^{\mu\nu} \) & \( Q^{\mu\nu} \) are projections being orthogonal to each other. The photon propagator then acquires the following form:

\[ G_{\mu\nu}(k_0, k) = \frac{-i P_{\mu\nu}}{k^2 - \pi_T(k_0, k) + i \varepsilon} + \frac{-i Q_{\mu\nu}}{k^2 - \pi_L(k_0, k) + i \varepsilon} \]

Radiative corrections in QED

- Electron/positron self-energy

Loop diagrams in case of the fermion propagator leads to

\[
\frac{i}{p - m + i\varepsilon} \quad \Rightarrow \quad \frac{i}{p - m - \Sigma(p) + i\varepsilon},
\]

where, at the leading order of the approximation, it holds that

\[-i\Sigma(p) = \begin{tikzpicture}
    
    \draw[thick,->] (0,0) -- (1,0) node[midway,above] {$p$};
    \draw[thick] (0.5,0) circle (0.2) (0.5,0) circle (0.4);
    \draw[thick] (0.2,0) -- (0.3,0) (0.3,0) -- (0.4,0) (0.4,0) -- (0.5,0) (0.5,0) -- (0.6,0) (0.6,0) -- (0.7,0) (0.7,0) -- (0.8,0) (0.8,0) -- (0.9,0) (0.9,0) -- (1,0);
\end{tikzpicture} + \mathcal{O}(\alpha^2)\]

Small black holes: One-loop effects

How small?

A black hole is said to be small if

\[ T_H \gg m_e \]

In this case, the electron/positron field can be considered as being effectively massless (hard-thermal-loop approximation).

In other words, small black holes have a mass in the following range:

\[ 10^{10} \text{ g} \lesssim M \ll 10^{16} \text{ g} , \]

where the lower bound is to have a quasi-equilibrium approximation.
Small black holes: One-loop effects

Reminder: Isotropic neutral electron-positron plasma at $T \gg m_e$

\[ \phi(r) = \frac{q}{4\pi r} e^{-r/r_D} \]

\[ r_D \equiv \frac{1}{\sqrt{2m_\gamma}} \]

\[ k_0^2 - k^2 = m_\gamma^2 \]

\[ m_\gamma^2 \equiv \frac{1}{6} e^2 T^2 \]

\[ \omega_p = \frac{1}{9} e^2 T^2 \]

Small black holes: One-loop effects

- Debye-like screening: Spacetime with an evaporating black hole
Small black holes: One-loop effects

- Debye-like screening: Spacetime with an evaporating black hole

Detector

\[ \phi(r) \]

Black Hole
Small black holes: One-loop effects

- Debye-like screening: Spacetime with an evaporating black hole

\[ r \ll R \]
\[ L \ll R \]
\[ R \gg r_H \]
Debye-like screening: Spacetime with an evaporating black hole

In the far-from-horizon region, a point-like charge $q$ at rest has the following (modified due to $\pi_L(0, k) \neq 0$) electrostatic potential:

$$\phi(r) \approx \frac{q}{4\pi r} e^{-r/r_D} \quad \text{with} \quad r_D \equiv \frac{2}{\pi m_\gamma},$$

where

$$m_\gamma^2 \equiv \frac{1}{6} e^2 T_L^2 \sim \alpha T_H^2 \left(\frac{r_H}{R}\right)^2 \sim \frac{\alpha}{R^2}.$$
Small black holes: One-loop effects

- Modified photon dispersion relation

\[ r_H \ll R \]

\[ \lambda_\gamma \ll R \]

\[ \lambda_\gamma \ll r_H \]
Small black holes: One-loop effects

- Modified photon dispersion relation

Due to $\pi_T(k_0, k) \neq 0$ in the limit $|k| \to k_0$, the photon propagator gets its pole shifted, namely

$$k_0^2 - k^2 = m_\gamma^2 \approx \frac{9\alpha}{128\pi} \times \begin{cases} +1/R^2, & R \gg r_H, \\ -1/r_H^2, & R \sim r_H, \end{cases}$$

where $R$ is a distance to the black-hole centre.

arXiv:hep-th/1703.05078
The pole structure of the electron propagator is also modified:

\[ p_0^2 - p^2 \approx m_e^2 + \frac{27\alpha}{256\pi} \times \begin{cases} 
  +\frac{1}{R^2}, & R \gg r_H, \\
  -\frac{1}{r_H^2}, & R \sim r_H,
\end{cases} \]

where \( R \) is a distance to the black-hole centre.
Is the Debye-like screening effect testable?

There are two problems:

- It is far from clear whether small black holes exist in nature.

- If existent, it is unclear how long one needs to wait to have at least one of these in the neighbourhood of earth at the distance not larger than roughly $10^3$ km.
Concluding remarks

Is the Einstein causality violated in the near-horizon region?

In general, a negative mass-squared term implies either instability or causality violation.

The instability is caused by modes with $|k| \leq m_\gamma \ll 1/r_H$, but our approximation holds only for modes with $|k| \gg 1/r_H$.

It means that there is a natural IR cutoff being of order $1/r_H$. A posteriori we know that modes with momentum higher or equal than local space-time curvature is immaterial for particle physics.

Thus, it seems that locality is violated near the event horizon of evaporating black holes.