Aharonov-Bohm Protection of Black Hole’s Skyrmion Hair

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based on

▶ ”Skyrmion black hole hair: Conservation of baryon number by black holes and Observable manifestations” (hep-th 1605.00543, Nucl. Phys. B913 (2016))
▶ ”Aharonov-Bohm protection of black hole’s baryon/skyrmion hair” (hep-th 1611.09370, Phys. Lett. B768 (2017))

Introduction: Black Hole Hair

- No hair conjecture (Wheeler, Ruffini, 1971)
- No hair theorems (Bekenstein, Teitelboim, Hartle, ...)
- Black holes with classical hair (Volkov, Galtsov and many others), many known
  - Many of them are known to be dynamically unstable
  - Topology seems to be a necessary condition for stability (in Lyapunov sense) in the asymptotically-flat case
- Black holes with Aharonov-Bohm quantum hair:
  - massless axions (Bowick, Giddings, Harvey, Horowitz, Strominger, 1988)
  - massive axions (Allen, Lahiri, 1990)
  - discrete ($Z_N$) gauge symmetry (Krauss, Wilczek, Preskill, Coleman, 1989-1992)
  - massive spin 2 (Dvali, 2006)
- Recent discussions on other types of black hole hair

**In this talk:** Skyrmion black hole hair both classical (known for a long time) and Aharonov-Bohm type (new)
Skyrmion Black Hole Hair I: Reminder about skyrmions (Skyrme 1961/1962, Witten 1983)

\[ \mathcal{L}_{\text{sky}} = -\frac{F_\pi^2}{4} \text{Tr} \left( U^+ \partial_\mu UU^+ \partial^\mu U \right) + \frac{1}{32e^2} \text{Tr} \left( \left[ \partial_\mu UU^+, \partial_\nu UU^+ \right]^2 \right) \]

(masses pions, two quark flavors)

\[ U = e^{\frac{i}{F_\pi} \sum_a \pi_a \sigma_a} \]

solitonic configurations: \( \frac{\pi_a}{F_\pi} = F(r)n_a, F(0) = B_\pi, F(\infty) = 0 \)

\[ B = \int_{R^3} J_0, \quad J_\mu = \frac{-1}{24\pi^2} \epsilon_{\mu\nu\alpha\beta} \text{Tr} \left( U^+ \partial_\nu UU^+ \partial_\beta UU^+ \partial_\alpha U \right) \]

\[ L = \frac{1}{F_\pi e}, \quad M_S = \frac{F_\pi}{e} \]

Classical skyrmions identifiable with baryons for large \( N_C \) in a world with only pions and no other meson degrees of freedom
Skyrmion Black Hole Hair II: Classical

Classical black hole skyrmion hair (Luckock, Moss, Droz, Heusler, Straumann, Bizon, Chmaj, Shiiki, Sawado, ...)

\[ G_{\mu\nu} = 8\pi G_N T_{\mu\nu}, \quad T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} \mathcal{L}_{\text{sky}})}{\delta g^{\mu\nu}} \]

\[ ds^2 = N^2(r) \left(1 - \frac{2M(r)G_N}{r}\right) dt^2 - \left(1 - \frac{2M(r)G_N}{r}\right)^{-1} dr^2 - r^2 d\Omega^2 \]

Numerical solutions which have an event horizon and non-vanishing profile function \( F(r) \) (with non-vanishing topological charge) exist, skyrmion hair can be detected e.g. via classical scattering experiments, parameter domain of these black hole solutions:

- skyrmion not itself a black hole
- event horizon located inside the soliton core

\[(\alpha \equiv 4\pi G_N F^2)\]
Skyrmion Black Hole Hair III: Quantum (Aharonov-Bohm)

\[ J_\mu = \star dS \text{ where (for } B = 1) \]
\[ S_{\mu\nu} = -\frac{1}{4\pi^2} \left( F(r) - \frac{1}{2} \sin(2F(r)) - \pi \right) \partial_{[\mu} \cos \theta \partial_{\nu]} \phi \]

Thus \( \int_{S^2} S \) is skyrmion charge defined at infinity!

expression for \( S \) also exists in general without making any (hedgehog) ansatz!

Black Hole Solution of Einstein Skyrme equations:
\[ ds^2 = \left( 1 - \frac{2MG_N}{r} \right) dt^2 - \left( 1 - \frac{2MG_N}{r} \right)^{-1} dr^2 - r^2 d\Omega^2 \]
\[ S = \frac{1}{4\pi} \sin \theta d\theta \wedge d\phi \] (in complete analogy to e.g. axion black holes)

This is black hole with skyrmion charge \( B = 1 \) (can generalize for arbitrary \( B \)). Cannot be detected classically, so can this charge be measured somehow?
Detecting AB-type Skyrmion Black Hole Hair

Couple two form $S_{\mu\nu}$ to a probe string with a coupling constant $g$; action =

$$g \int d^4x \int d^2\sigma \partial_a X^\mu \partial_b X^\nu \epsilon^{ab} \delta^{(4)}(x - X) S_{\mu\nu} = g \int S_{\mu\nu} dX^\mu \wedge dX^\nu$$

Induces Aharonov-Bohm phase shift $\Delta \Phi = 2\pi g \rightarrow$ measurable if $g$ is non-integer
Baryon Number Conservation by Black Holes?

"Standard" folklore of folk theorems: Global charges such as baryon number are incompatible with semi-classical black hole physics; Assumptions involved:

- No hair
- Thermal evaporation of a black hole which initially swallowed a baryon/skyrmion down to Planck size

Loophole: Skyrmion/Baryon Hair exists!

- Natural possibility for dynamical process: Baryon/Skyrmion emerges as a classical skyrmion hair when black hole reached some critical size \( L \), shrinks further down inside of the skyrmion/baryon
Aharonov-Bohm protection of black hole's baryon/skyrmion hair. Questions?

Dear Alexander Gußmann,

Greetings from Journal of Hair Therapy and Transplantation!!!

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We have gone through your profile and articles which you have published. Your area of interest is suitable for our journal and it will help you also to broaden your scientific research. It's a great honour for me to consider you as potential author; based on your previous exceptionally well written article entitled "Aharonov–Bohm protection of black hole's baryon/skyrmion hair". It was an objective look at the issue, so we would like to invite you to contribute a manuscript for consideration and publication in upcoming Issue of Journal of Hair therapy and Transplantation (Volume 7 Issue 1).