Quantum corpuscular corrections to the Newtonian potential

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Outline of the talk

- Motivation: Black Holes & the road to QG
- Matter matters! (Baryons & gravitons)
- Classical & Quantum corrections to the Newtonian potential
- Outlook
Motivation: Black Holes & the road to QG
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Horizon $r = R_H = 2 G_N M(t, r)$

Trapped surface $g^{ij} \nabla_i r \nabla_j r = 0$

Staticity

Misner-Sharp mass

Strong gravity scale
Motivation: Black Holes & the road to QG

Trapped surface
\[ g^{ij} \nabla_i r \nabla_j r = 0 \]

Staticity

Horizon
\[ r = R_H = 2 G_N M(t, r) \]

Misner-Sharp mass

Strong gravity scale

... but nature is quantum!!!
\[ \Delta x \Delta p \geq \hbar \]

Compton wavelength
\[ \lambda_M = \frac{\hbar}{M} \]

Q(FT) scale
Motivation: Black Holes & the road to QG

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Trapped surface: \( g^{ij} \nabla_i r \nabla_j r = 0 \)

Staticity

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Compton wavelength

\( \Delta x \Delta p \geq \hbar \)

\( \hbar = l_P m_p \)

\( G_N = \frac{l_P}{m_p} \)

QUANTUM BLACK HOLES

\( M \geq m_P \)

Q(FT) scale

… but nature is quantum!!!
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Trapped surface:

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Horizon:

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Staticity:

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\[ \Delta x \Delta p \geq \hbar \]

... but nature is quantum!!!

Strong gravity scale

\[ \hbar = l_p m_p \]

\[ G_N = \frac{l_p}{m_p} \]

Counterproposal

Quantum Black Holes

\[ N m \geq m_p \]

Q(FT) scale
Matter matters! (Baryons & gravitons)
R.Casadio, A.G. and A.Giusti, PLB 763 (2016) 337

BH = self-sustained BEC of gravitons

\[ m = -\epsilon_G \quad \text{Effective mass} \]

\[ M \approx Nm \]

\[ R_H \approx \sqrt{N} \ell_p \]

\[ m \approx \frac{m_p}{\sqrt{N}} \]

\( N \gg 1 \)

\[ U \approx -m \frac{\ell_p}{m_p} r \sim \lambda_m \sim \frac{\ell_p m_p}{m} \approx -mN \left( \alpha \sim \frac{m^2}{m_p^2} \right) \]

\[ E \approx m + U \approx 0 \quad \text{“marginally bound”} \]

\[ \Delta E = m \quad \text{Hawking quantum} \]

\[ \Gamma \sim \frac{1}{N^2} N^2 \frac{1}{\sqrt{N} \ell_p} \]

Hawking evaporation:

\[ \dot{M} \approx m \frac{\dot{N}}{\sqrt{N}} \sim -m \frac{\Gamma}{\sqrt{N}} \sim -\frac{m_p^3/\ell_p}{M^2} \]
Matter matters! (Baryons & gravitons)
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“Energy balance”: \[ H \equiv H_B + H_G = M \]

\[ H_B(\infty) = \mu N_B \simeq M \]

\[ H = M + K_B(R) + U_{BG}(R) + U_{BB}(R) + U_{GG}(R) \]

\[ U_{BG}(R) \simeq N_B \mu \phi_N(R) \simeq -N_B \mu \frac{\ell_p M}{m_p R} = -\frac{M^2 \ell_p}{m_p R} \]

\[ U_{BB}(R) \simeq N_G \epsilon_G(R) \]

Newtonian correction!

\[ \epsilon_G \simeq -\frac{\ell_p}{R} m_p \]

\[ U_{GG}(R) \simeq N_G \epsilon_G(R) \phi_N(R) \simeq N_G \frac{M \ell_p^2}{R^2} \]

\[ U_{GG}(R_H) \simeq -U_{BG}(R_H) \simeq M \]

“marginal bound” recovered!

\[ R_H \simeq \sqrt{N} \ell_p \]

\[ m \simeq \frac{m_p}{\sqrt{N}} \]
Post-Newtonian effective theory

Schwarzschild

$$ds^2 = - \left(1 - \frac{2M}{\tilde{r}}\right) d\tilde{t}^2 + \left(1 - \frac{2M}{\tilde{r}}\right)^{-1} d\tilde{r}^2 + \tilde{r}^2 d\Omega^2$$

From area to distance:

$$dr = \frac{d\tilde{r}}{\sqrt{1 - \frac{2M}{\tilde{r}}}} \quad (... \text{irrelevant})$$

Static proper time:

$$dt = \left(1 - \frac{2M}{\tilde{r}}\right)^{1/2} \tilde{t}$$

Radial geodesic fall:

$$\frac{d^2\tilde{r}}{d\tilde{t}^2} = -\frac{M}{\tilde{r}^2}$$
Post-Newtonian effective theory

**GR recap**

Schwarzschild

\[ ds^2 = -\left(1 - \frac{2M}{\tilde{r}}\right) \, d\tilde{t}^2 + \left(1 - \frac{2M}{\tilde{r}}\right)^{-1} \, d\tilde{r}^2 + \tilde{r}^2 \, d\Omega^2 \]

- **Radial geodesic fall:**
  \[ \frac{d^2 \tilde{r}}{d\tilde{t}^2} = -\frac{M}{\tilde{r}^2} \]

- **From area to distance:**
  \[ dr = \frac{d\tilde{r}}{\sqrt{1 - \frac{2M}{\tilde{r}}}} \quad (\ldots \text{irrelevant}) \]

- **Static proper time:**
  \[ dt = \left(1 - \frac{2M}{\tilde{r}}\right)^{1/2} \, d\tilde{t} \]

**Conditions:**
- Weak field
- Static non-relativistic motion

**Standard post-Newtonian from isotropic metric**

\[ \frac{d^2 r}{dt^2} \sim -\frac{d}{dr} \left( -\frac{M}{r} + \frac{M^2}{r^2} \right) \]

Watch out!
- Different parametrisations of \( t \)
- ...but same functional form
Post-Newtonian effective theory

Einstein-Hilbert action:

\[ S_{EH} = \int d^4x \sqrt{-g} \left( \frac{m_p}{16 \pi \ell_p} \mathcal{R} + \mathcal{L}_M \right) \]

\[ \mathcal{L}_M = \rho \]

1) Weak field:

2) Static non-relativistic motion:

3) De Donder gauge:

4) Fierz-Pauli and some guessing ...

\[ g_{\mu\nu} = \eta_{\mu\nu} + \epsilon \ h_{\mu\nu} \]

\[ h_{\mu\nu} \approx h_{00} = -2 \ V \]

\[ 0 = 2 \partial^\mu h_{\mu\nu} - \partial_\nu h \approx \partial_t V \]
Einstein-Hilbert action:

\[ S_{EH} = \int d^4 x \sqrt{-g} \left( \frac{m_p}{16 \pi \ell_p} \mathcal{R} + \mathcal{L}_M \right) \]

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1) Weak field:
2) Static non-relativistic motion:
3) De Donder gauge:
4) Fierz-Pauli and some guessing ...

Post-Newtonian effective theory

Effective scalar action:

\[ S[V] = 4 \pi \int \epsilon dt \int_0^\infty r^2 dr \left\{ \frac{m_p}{8 \pi \ell_p} V \Delta V - \rho V + \frac{\epsilon}{2} \left[ \frac{m_p}{4 \pi \ell_p} (V')^2 + V \rho \right] V \right\} \]
Post-Newtonian effective theory

Newtonian potential:

\[ H[V_N] = -L[V_N] = 4\pi \int_0^\infty r^2 dr \left( -\frac{m_p}{8\pi \ell_p} V_N \Delta V_N + \rho V_N \right) \]

Field equation: \[ \Delta V_N = 4\pi \frac{\ell_p}{m_p} \rho \]

Purely Newtonian potential energy

\[ U_N(r) = 2\pi \int_0^r \tilde{r}^2 d\tilde{r} \rho(\tilde{r}) V_N(\tilde{r}) \]

Gravitational potential self-energy

\[ U_{GG} \sim \int r^2 dr J_V V \]

Potential energy density (aka static current):

\[ J_V(r) = \frac{1}{4\pi r^2} \frac{d}{dr} U_N(r) = -\frac{m_p}{8\pi \ell_p} \left[V_N'(r)\right]^2 \]
Post-Newtonian effective theory

Field equation:

\[ L[V] = -4\pi \int_0^\infty r^2 dr \left[ \frac{m_p}{8\pi \ell_p} (1 - 4q\Phi V) (V')^2 + q_B V \rho (1 - 2q\Phi V) \right] \]

\[
(1 - 4q\Phi V) \Delta V = 4\pi q_B \frac{\ell_p}{m_p} \rho (1 - 4q\Phi V) + 2q\Phi (V')^2
\]

Separate & solve:

\[ V(r) = V_{(0)}(r) + q\Phi V_{(1)}(r) \]

\[ \Delta V_{(0)} = 4\pi q_B \frac{\ell_p}{m_p} \rho \]

\[ \Delta V_{(1)} = 2 \left( V_{(0)}' \right)^2 \]

Potential energies

\[ U_{BG} = 2\pi q_B \int_0^\infty r^2 dr \rho \left[ V_{(0)} + q\Phi \left( V_{(1)} - 4V_{(0)}^2 \right) \right] \]

\[ U_{GG} = -3q\Phi \frac{\ell_p}{m_p} \int_0^\infty r^2 dr V_{(0)} \left( V_{(0)}' \right)^2 \]
Classical solutions

(Rough) Example: Point-like source

\[ \rho = M_0 \delta^{(3)}(x) = \frac{M_0}{4\pi r^2} \delta(r) \]

(0 + 1) potential:

\[ V \approx -q_B \frac{\ell_p M}{m_p r} + q_\Phi q_B \frac{\ell_p^2 M^2}{m_p^2 r^2} \]

(0 + 1) energies:

\[ U_{BG} \approx -q_B^2 \frac{\ell_p M_0 M}{2 m_p r_0} - q_B^3 q_\Phi \frac{3 \ell_p^2 M^3}{2 m_p^2 r_0^2} \]
\[ U_{GG} \approx q_B^3 q_\Phi \frac{3 \ell_p^2 M^3}{2 m_p^2 r_0^2} \]

[\delta(r) \rightarrow \delta(r - r_0)]

Arbitrary!
Post-Newtonian effective theory

Classical solutions

(Rough) Example: Point-like source

\[ \rho = M_0 \, \delta^{(3)}(x) = \frac{M_0}{4 \pi \, r^2} \, \delta(r) \]

(0 + 1) potential:

\[ V \simeq -q_B \, \frac{\ell_p M}{m_p \, r} + q_\Phi \, q_B \, \frac{\ell_p^2 M^2}{m_p^2 \, r^2} \]

(0 + 1) energies:

\[ U_{BG} \simeq -q_B^2 \, \frac{\ell_p \, M_0 \, M}{2 \, m_p \, r_0} - q_B \, q_\Phi \, \frac{3 \, \ell_p^2 \, M^3}{2 \, m_p^2 \, r_0^2} \]

\[ U_{GG} \simeq q_B^3 \, q_\Phi \, \frac{3 \, \ell_p^2 \, M^3}{2 \, m_p^2 \, r_0^2} \]

Post-Newtonian too large!

Arbitrary!

\[ \delta(r) \rightarrow \delta(r - r_0) \]

NO maximal packing for point source!
Post-Newtonian effective theory

More realistic case: homogeneous star

\[ \rho(r) = \frac{3 M_0}{4 \pi R^3} \Theta(R - r) \]

(0 + 1) energy:

\[ U(R) \simeq -q_B^2 \frac{3 \ell_p^2 M^2}{5 m_p R} + q_B^3 q_F \frac{249 \ell_p^2 M^3}{175 m_p^2 R^2} \]
Post-Newtonian effective theory

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The result holds also for other kinds of sources (e.g. Gaussian, etc.)

“marginally bound” for \( R \approx 1.2 R_H \)
Post-Newtonian effective theory

Quantum toy model: massless scalar field

Field equation: \( \Box \Phi = 0 \rightarrow \hat{\Phi}(t, r) = \ell_p \int_0^\infty \frac{k^2 \, dk}{2 \pi^2 \sqrt{2 \, k}} \, j_0(k \, r) \left( \hat{a}_k \, e^{i \, k \, t} + \hat{a}_k^\dagger \, e^{-i \, k \, t} \right) \)

Commutators: \( [\hat{a}_p, \hat{a}_k^\dagger] = \frac{2 \, \pi^2}{k^2} \, \delta(p - k) \)

Vacuum: \( \hat{a}_k \, |0\rangle = 0 \)

Newtonian ground state: \( \hat{a}_k \, |g\rangle = e^{-i \, k \, t} \, g_k \, |g\rangle \)

Coherent!

\[ g_k = \sqrt{\frac{k}{2} \, \tilde{V}_c(k)} = -\frac{q_B \, \bar{\rho}(k)}{m_p \, \sqrt{2 \, k^3}} \]

Normalisation: \( \langle g \, |g\rangle = 1 \)

\[ \langle g \, |\hat{\Phi}(t, r) \, |g\rangle = \int_0^\infty \frac{k^2 \, dk}{2 \, \pi^2} \, j_0(k \, r) \, \tilde{V}_c(k) = V_c(r) \]
Post-Newtonian effective theory

Normalisation ~ occupation number

\[ |g\rangle = e^{-\frac{N_G}{2}} \exp \left\{ \int_0^\infty \frac{k^2 \, dk}{2 \pi^2} g_k \, \hat{a}_k^\dagger \right\} |0\rangle \]

\[ N_G = \int_0^\infty \frac{k^2 \, dk}{2 \pi^2} g_k^2 = \langle g | \int_0^\infty \frac{k^2 \, dk}{2 \pi^2} \hat{a}_k^\dagger \hat{a}_k | g \rangle \]

Typically:

\[ N_G \sim \frac{M^2}{m_p^2} \ln \left( \frac{\Lambda}{k_0} \right) \sim \frac{M^2}{m_p^2} \ln \left( \frac{R_\infty}{R} \right) \]

\[ \frac{dN_G}{N_G} \sim \frac{dM}{M} - \frac{1}{\ln(R_\infty/R)} \frac{dR}{R} \]

UV cut-off

Size of static field

IR cut-off

Source size

Dynamics...
Post-Newtonian effective theory

Post-Newtonian corrections:

\[ \Delta V_{(1)} = 2 \frac{\ell_p}{m_p} \langle g' \right| \left( \hat{\Phi}' \right)^2 | g \rangle = 2 \left( V_{(0)}' \right)^2 + \mathcal{V}_0 \]

New coherent state:

\[ \hat{a}_k | g' \rangle \simeq g_k | g \rangle + \Phi g_k | g \rangle \]

\[ \sqrt{\frac{\ell_p}{m_p}} \langle g' | \hat{\Phi} | g' \rangle \simeq V_{(0)} + \Phi V_{(1)} \]

One-mode occupation:

\[ \bar{k} \sim R^{-1} \]

\[ \delta g_{\bar{k}} \sim -\ell_p \bar{k}^{5/2} g_{\bar{k}}^2 \]

Typically:

\[ \delta g_{\bar{k}} \sim \frac{R_H}{R} g_{\bar{k}} \]

OK for large source!
Post-Newtonian corrections:

\[ \Delta V_{(1)} = 2 \frac{\ell_p}{m_p} \langle g | (\hat{\Phi}')^2 | g \rangle = 2 \left( V_{(0)}' \right)^2 + \chi_0 \]

New coherent state:

\[ \hat{a}_k |g'\rangle \simeq g_k |g\rangle + q_\Phi \delta g_k |g\rangle \]

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OK for large source!

For black hole formation?
Outlook

• Quantum post-Newtonian corrections:
  • Phenomenological consequences for (neutron) stars
  • Phenomenological consequences for BH formation

• Thermal fluctuations:
  • Quantum matter effects
  • Hawking radiation

• Quantum resolution of inner BH structure
Thank you!